On the non-minimal coupling of Riemann-flat Klein-Gordon Fields to Space-time torsion

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Abstract

The energy spectrum of Klein-Gordon particles is obtained via the non-minimal coupling of Klein-Gordon fields to Cartan torsion in the approximation of Riemann-flatness and constant torsion.When the mass squared is proportional to torsion coupling constant it is shown that the splitting of energy does not occur.I consider that only the vector part of torsion does not vanish and that it is constant.A torsion Hamiltonian operator is constructed.The spectrum of Klein-Gordon fields is continuos.

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I Introduction

Earlier Figueiredo,Damião Soares and Tiomno (FTD) [1, 2, 3] investigated the gravitational coupling of Klein-Gordon and Dirac fields to matter vorticity and torsion in some cosmological models.Some years earlier H.Rumpf [4] considered a particular case of that problem namely a Riemann-flat spectrum for Dirac particles in the case of constant torsion and electromagnetic fields.More recently Claus Lämmerzhal [5] have considered the case of the coupling of space-time torsion to the Dirac equation showing the effects on the energy levels of atoms which can be tested by the Hughes-Drever experiments.In this paper he was able to place a limit on the axial torsion testing the anisotropy of anomalous spin couplings and mass.Yet more recently I.L.Shapiro [6] and Bagrov, Buchbinder and Shapiro [7] considered the possibility of testing torsion theories in the low energy limit by making use of a non-Minimal coupling Lagrangian with torsion and scalar fields.At this point is important to note that in references [1, 2, 3, 4, 5] the authors have considered that the torsion did not propagate or in other words they considered to be dealing with the Einstein-Cartan gravity torsion does not propagate.Here on the contrarywe consider the general case where torsion propagates and besides we assume a non-minimal coupling where the Klein-Gordon scalar fields couple with torsion, contrary to the (FDT) approach where only Dirac fields couple to torsion through the spinorial connection.This is also the point of view addopted by Carroll and Field [8] who showed that for a wide class of models the only modes of the torsion tensor which interact with matter are either a massive scalar or a massive spin-1 boson. In fact the Lagrangian we addopt here is nothing but a slight modification of their Lagrangian

$$L = a\partial_\mu T_\nu \partial^\mu T^\nu + b(\partial_\mu T^\mu)^2 + cT_\mu T^\mu \tag{1}$$

where T^μ is the torsion pseudo-vector.Our Lagrangian is given by

$$L_T = L = \partial_\mu \phi \partial^\mu \phi + \partial_\mu T^\mu + T_\mu T^\mu - m^2\phi^2 \tag{2}$$

This Lagrangian can in fact be obtained from the following Lagrangian

$$L = \partial_\mu \phi \partial^\mu \phi + sR\phi^2 - m^2\phi^2 \tag{3}$$

Where s represents the torsion coupling and the last term is the potential energy term and the other terms in equation (3) are obtained by considering

that the scalar curvature R is Riemann- flat and given by

$$R = \partial_\mu T^\mu + T_\mu T^\mu \tag{4}$$

To simplify matters we shall from now on suppress indices.Thus variation of Lagrangian (2) with respect to torsion and the scalar field ϕ yields respectively the equations

$$T = -\frac{\partial\phi}{\phi} \tag{5}$$

and

$$\partial^2\phi + s\partial\phi - \frac{m^2}{2}\phi = 0 \tag{6}$$

Let us now solve this system for the case of plane symmetry similar to the case of domain walls in Riemann-Cartan space-time [9]).In this case the above equations may be written in the form

$$\phi^{"} + s\phi^{`} - \frac{m^2}{2}\phi = 0 \tag{7}$$

where the upper primes denote derivatives with respect to coordinate-z.Note that the second term in equation (7) represents a damping like a friction in domain walls,thus one may say that torsion introduces a sort of friction into the problem.Discrete spectrum can be found for example in refence [1] in the case of the Gödel cosmological model.The LHS of equation (7) can be written in operator form as

$$\hat{H}\phi = (\hat{H}_0 + \hat{H}_{torsion})\phi \tag{8}$$

where $\hat{H}_0$ is the basic Klein-Gordon Hamiltonian operator and $\hat{H}_{torsion}$ is the Hamiltonian torsion operator given by

$$\hat{H}_{torsion} = s\frac{\partial}{\partial z} \tag{9}$$

A similar operator but much more involved have been constructed by Lämmerzhal [5].To solve (6) we make use of a simple ansatz

$$\phi = e^{cz} \tag{10}$$

Substitution of (10) into (6) yields the following constraint equation

$$c^2 + sc + m^2 = 0 \tag{11}$$

which is an algebraic very simple equation which yields

$$c = \frac{-s \pm (s^2 - 4m^4)^{\frac{1}{2}}}{2} \tag{12}$$

substitution of (12) into (10) yields the scalar field

$$\phi = e^{\frac{-s \pm (s^2 - 4m^4)^{\frac{1}{2}}}{2} z} \tag{13}$$

one may calculate from (13) the following energy

$$\epsilon = |\phi^{'}|^2 + V(\phi) \tag{14}$$

substitution of (13) into (14) yields

$$\epsilon = |\frac{-s \pm (s^2 - 4m^4)^{\frac{1}{2}}}{2}|^2 e^{cz} \tag{15}$$

From this last expression one notes immediatly that there is a splitting of the spectral lines of the Klein-Gordon fields always that torsion does not coincide with the double of the mass squared.The physical constraint required here unfortunatly makes the task of measuring torsion directly since at least in astrophysical stellarobjects the mass is always much higher the torsion imagine the mass squared.Nevertheless indirect measurements can be sugested.Although there is a double splitting in the energy one must observe that this spectrum is continuos.In reference (1) a similar situation appears, nevertheless in their case vorticity plays the role of mass here.In other words is the simultaneous presence of torsion and mass here that produces the splitting of the energy levels.It is also important to note that the torsionless case is not allowed here since in this case the energy would be a complex number.Discrete spectrum can also be found in reference [1] in the case of the Gödel cosmological model.Yet in reference [7] Bagrov et al. obtained a double splitting of the spectral lines of the Hydrogen atom by considering the low energy limit of torsion in the Schrödinger equation.Also in their case the splitting is a pure torsion effect and does not depend on the magnetic field.A more detailed analysis of these experiments may appear elsewhere.Besides a small change in the potential in our Lagrangian would allows to investigate gravitational torsion kinks or domain walls.

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References

[1] B.D.B.Figueiredo,I.Damião Soares and J.Tiomno,(1991) Gravitational Coupling of Klein-Gordon and Dirac Particles to Matter Vorticity and Space-Time Torsion,CBPF-NF-014/91,Rio de Janeiro.

[2] B.D.B.Figueiredo,D.Sc.Thesis,Centro Brasileiro de Pesquisas Fisicas-CBPF,(1987) in Portuguese.

[3] B.D.B.Figueiredo,I.Damião Soares and J.Tiomno,Phys.Lett.A137,99 (1989)

[4] H.Rumpf,Gen.Rel.and Grav.10,647(1979).

[5] C.Lämmerzhal,Phys.Lett.A 228(1997)223.

[6] I.L.Shapiro(1994)Mod.Phys.Lett.9 A 729.

[7] V.G.Bagrov,I.L.Buchbinder and I.L.Shapiro, On the possible experimental manifestations of the torsion field at low energies,gr-qc preprints,Los Alamos electronic archives.

[8] S.M.Carroll and G.B.Field,Phys.Rev.D 50(1994) ,6,3867.

[9] L.C.Garcia de Andrade,(1998),On non-Riemannian domain walls,Gen.Rel.Grav.J.in press.